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Simple method for transient response of gas-to-gas cross-flow heat exchangers with neither gas mixed

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Abstract—The two-dimensional transient response of gas-to-gas cross-flow heat exchangers is investigated analytically by the method of a single Laplace transform for arbitrary time variations of the primary fluid inlet temperature. Analytical solutions for transformed temperature distributions of the core wall and both fluids are presented in the form of a power series with the heat capacity ratios, number of transfer units, heat transfer resistance and flow capacitance ratios. The transformed temperatures of the core wall and both fluids are easily inverted to the physical quantities by using the numerical inversion scheme of the Laplace transform. As compared with other analytical solutions, the present method has good accuracy and efficiency.

INTRODUCTION

THE TRANSIENT response of cross-flow heat exchangers is of increasing interest in many industrial fields, such as aircraft gas turbines, air-conditioning systems, phosphoric acid fuel cell power units and dirty gas applications. It is important to study the dynamic behaviours of a cross-flow heat exchanger in order to obtain a correct and reliable design, control and operation for reducing the energy waste and prevent danger and expensive off-lines. A fast mathematical-simulation scheme (with acceptable engineering accuracy) capable of predicting the transient response of the system is always desired for correct design of heat exchangers.

A considerable effort has been made in investigating the steady-state solutions for temperature profiles of cross-flow heat exchangers [1]. However, the transient response of cross-flow heat exchangers has received very little attention owing to its complexity. Some authors [2–7] proposed numerical solutions of these problems in which the fluid-to-wall capacity ratio is equal to zero or the wall capacitance is large. Dusinberre [3] presented the first paper to deal with the transient behaviours of gas-to-gas cross-flow heat exchangers with both fluids unmixed. He applied the finite difference method to determine transient solutions of gas temperatures but only one specific case was considered. Afterward, Yamashita *et al.* [4] also calculated the outlet temperature responses of the cross-flow heat exchangers without fluids mixed by using the finite difference method. They further investigated the effects of an initial condition and various parameters on the outlet temperature response. However, the application of the finite difference method to such problems has a severe limitation on the step-size. On the other hand, this method will tend to require an excessive amount of computer time when only the outlet temperature responses at a specific time or at a specific position are given. Myers *et al.* [5] employed an integral technique to the analysis of gas-to-gas cross-flow heat exchanger with one mixed fluid. They obtained an approximate solution of the outlet fluid temperatures for a unit step change in the inlet temperature of the mixed fluid. Romie [6] gave the transient mixed mean temperatures of the two gases leaving a cross-flow heat exchanger for a unit step change in the entrance temperature of either gas. Solutions were obtained by the double Laplace transforms method and were applied to the single-pass cross-flow exchangers without mixing of gases. Spiga and Spiga [2] and Gvozdenac [7] investigated the two-dimensional transient behaviours of gas-to-gas cross-flow heat exchangers with arbitrary initial and inlet conditions by using the two- and threefold Laplace transform, respectively. The complexity of the inversive task of the two- and threefold Laplace transform is known to all. On the other hand, it is difficult to invert the transformed temperatures of the core wall and both fluids to the physical quantities when the two- and threefold Laplace transform are applied. In addition, it can be found that none of them [2, 6, 7] employed any numerical scheme in inverting the physical domain. In cases with heavier restrictions, results are expressed in more or less explicit formulas which may or may not be convenient to compute. However, in more general cases, the results in the transformed domain are so complex that it is difficult to invert them to the physical quantities.

The present work analyses the transient behaviours of gas-to-gas cross-flow heat exchangers by using the method of the single Laplace transform with respect to time. The aim of this work is to develop a straightforward computer code for such problems. The trans-

NOMENCLATURE

A^*	heat transfer surface
c	specific heat
E	flow capacitance ratio
h	heat transfer coefficient
L	exchanger length
M	mass of exchanger
m	mass flow rate
N	dimensionless exchanger length
NTU	number of transfer units
R	heat transfer resistance ratio
s	Laplace transform parameter
t	dimensionless time variable
T	dimensionless temperature
$\tilde{T}$	transformed dimensionless temperature
x, y	dimensionless space variables.

Greek symbols

ξ, ζ	space variables
τ	time variable.

Subscripts

a	prime fluid
b	secondary fluid
w	solid wall.

formed temperatures expressed in the form of a power series are specialized in order to describe step, ramp and exponential responses. Owing to these transformed temperatures expressed in the form of a power series with regularization, it is evident that they are easily written in a computational program. The transformed temperatures of the core wall and both fluids in the present work are easily inverted to the physical quantities using the numerical inversion method of the Laplace transform proposed by Honig and Hirdes [8]. It can be found from the present analysis that the temperature responses of the core wall and both fluids at a specific time and position can be calculated using the present technique without any difficulty. For this case, the present method has considerable savings in computer time. In addition, the power series in the present work has a fast rate of convergence. A comparison of the present solutions and those given by Spiga and Spiga [2] is made. No difference between them is found. This conclusion shows that the present method has good accuracy and efficiency.

ANALYSIS

The dynamic response of cross-flow heat exchangers with walls separating the two fluid streams is investigated. Stream 'a' flows through a set of tubes or plates, arranged in a bank, however stream 'b' threads its way through the spaces at right angles to the bank. The mathematical model is developed resorting to the simplifying assumptions as follows [2, 6, 7]:

(a) neither fluid is mixed;

(b) the physical properties and the fluid capacity rates are independent of time, position and temperature;

(c) the thermal conductances on both sides are constant and inclusive of wall thermal resistance and fouling;

(d) the exchanger shell or shroud is adiabatic;

(e) the fluid velocity is constant in each flow passage;

(f) conduction through the fluid is negligible;

(g) heat transfer rate per unit area and surface configurations are constant;

(h) the heat generation and viscous dissipation within the fluids are negligible;

(i) the ratios of the thermal capacities of both fluids to the core wall thermal capacity are negligible, i.e. it is typical for gas-to-gas exchange units.

The dimensionless space and time-independent variables are defined for generating the equations of such problems as

$$t = \frac{(hA^*)_a}{Mc_w}\tau \tag{1a}$$

$$x = \frac{(hA^*)_a \xi}{(mc)_a L_a} \tag{1b}$$

$$y = \frac{(hA^*)_b \zeta}{(mc)_b L_b}. \tag{1c}$$

Then, applying the energy equation to both fluids and the wall, we have three simultaneous partial differential equations in the coordinate system [2]

$$\frac{\partial T_w}{\partial t} + (1+R)T_w = T_a + RT_b \tag{2a}$$

$$\frac{\partial T_a}{\partial x} + T_a = T_w \tag{2b}$$

$$\frac{\partial T_b}{\partial y} + T_b = T_w \tag{2c}$$

for $t \geqslant 0$, $0 \leqslant x \leqslant N_a$ and $0 \leqslant y \leqslant N_b$.

The corresponding initial and inlet conditions of equations (2) are given as

$$T_w(x, y, 0) = T_a(x, y, 0) = T_b(x, y, 0) = 0 \tag{3a}$$

$$T_a(0, y, t) = \varphi(t) \tag{3b}$$

$$T_b(x, 0, t) = 0 \tag{3c}$$

where the dimensionless physical parameters R, E, N_a, N_b and NTU are defined respectively as

$$R = \frac{(hA^*)_b}{(hA^*)_a} \tag{4a}$$

$$E = \frac{(mc)_b}{(mc)_a} \tag{4b}$$

$$N_a = \frac{(hA^*)_a}{(mc)_a} \tag{4c}$$

$$N_b = \frac{(hA^*)_b}{(mc)_b} \tag{4d}$$

$$NTU = \left\{(mc)_{\min}\left[\frac{1}{(hA^*)_a}+\frac{1}{(hA^*)_b}\right]\right\}^{-1} \tag{4e}$$

where it is seen from equations (4) that only three of them are independent.

The above model equations show that only the inlet condition of one fluid is perturbed. For simplicity, assume that such a perturbation is uniform in the plane of the inlet section.

MATHEMATICAL FORMULATION

The Laplace transform of $T\alpha(x, y, t)$ corresponding to the dimensionless time with the complex parameter s and its inversion formula are defined as

$$\tilde{T}\alpha(x,y,s) = L^{-1}\{T\alpha(x,y,t)\} = \int_0^\infty T\alpha(x,y,t)\,e^{-st}\,dt \tag{5a}$$

$$T\alpha(x,y,t) = L^{-1}\{\tilde{T}\alpha(x,y,s)\} \tag{5b}$$

where $\alpha = $ w, a, b. s is the Laplace transform parameter.

Taking the Laplace transform of equations (2), (3b) and (3c) with initial condition (3a) yields

$$\tilde{T}_w = \frac{1}{1+R+s}\tilde{T}_a + \frac{R}{1+R+s}\tilde{T}_b \tag{6a}$$

$$\frac{\partial \tilde{T}_a}{\partial x} + \tilde{T}_a = \tilde{T}_w \tag{6b}$$

$$\frac{\partial \tilde{T}_b}{\partial y} + \tilde{T}_b = \tilde{T}_w \tag{6c}$$

with the transformed boundary conditions

$$\tilde{T}_a(0,y,s) = \tilde{\varphi}(s) = \int_0^\infty \varphi(t)\,e^{-st}\,dt \tag{6d}$$

$$\tilde{T}_b(x,0,s) = 0. \tag{6e}$$

Substitution of equation (6a) into equations (6b) and (6c) can reduce the problem to a set of first-order partial differential equations as

$$\frac{\partial \tilde{T}_a}{\partial x} + A\tilde{T}_a = B\tilde{T}_b \tag{7a}$$

$$\frac{\partial \tilde{T}_b}{\partial y} + C\tilde{T}_b = D\tilde{T}_a \tag{7b}$$

where

$$A = 1 - \frac{1}{1+s+R} \tag{7c}$$

$$B = \frac{R}{1+s+R} \tag{7d}$$

$$C = 1 - \frac{R}{1+s+R} \tag{7e}$$

$$D = \frac{1}{1+s+R}. \tag{7f}$$

These equations are less complex than equations (2) since they do not depend upon the dimensionless time. Thus, the problem has now been reduced to solve equations (7a) and (7b) for $\tilde{T}_a$ and $\tilde{T}_b$.

The function $\tilde{T}_a(x, y, s)$ can be expressed in the form of a power series as described below

$$\tilde{T}_a(x,y,s) = \tilde{\varphi}(s) + \sum_{k=1}^{\infty} a_k(y,s)x^k. \tag{8}$$

It is seen that equation (8) satisfies the transformed boundary condition (6d).

Substituting equation (8) into equation (7a) gives

$$\tilde{T}_b(x,y,s) = \frac{1}{B}\left\{\sum_{k=1}^{\infty} a_k(y,s)kx^{k-1} + A\left[\tilde{\varphi}(s) + \sum_{k=1}^{\infty} a_k(y,s)x^k\right]\right\}. \tag{9}$$

Substituting equations (8) and (9) into equation (7b) and collecting the coefficients of successive powers of x yields the following form:

$$\left[\frac{da_1}{dy} + Ca_1 - (BD-CA)\tilde{\varphi}(s)\right] + \sum_{k=1}^{\infty}\left[\frac{da_{k+1}}{dy} + Ca_{k+1} - \frac{a_k}{k+1}(BD-CA) + \frac{A}{k+1}\frac{da_k}{dy}\right]x^k = 0. \tag{10}$$

The expression of equation (10) implies that the coefficients of all powers of x must vanish independently. The vanishing of the coefficient of x^k, $k \geqslant 0$, in equation (10), gives the following recurrence formula:

$$\frac{da_1}{dy} + Ca_1 = (BD-CA)\tilde{\varphi} \tag{11a}$$

and

$$\frac{da_{k+1}}{dy} + Ca_{k+1} = \frac{a_k}{k+1}(BD-CA) - \frac{A}{k+1}\frac{da_k}{dy}, \quad k \geqslant 1. \tag{11b}$$

Assume that the functions a_k, $k \geqslant 1$, can be expressed as

$$a_1(y,s) = f_1(s) + p_1(s)\,e^{-Cy}$$

$$a_2(y,s) = f_2(s) + (p_2(s) + q_2(s)y)\,e^{-Cy}$$

$$a_3(y,s) = f_3(s) + (p_3(s) + q_3(s)y + r_3(s)y^2)\,e^{-Cy}$$

$$a_4(y,s) = f_4(s) + (p_4(s) + q_4(s)y + r_4(s)y^2 + s_4(s)y^3)\,e^{-Cy} \quad (12)$$

By the direct substitution of equations (12) into equations (11) one obtains

$$f_1(s) = \frac{BD-CA}{C}\tilde{\varphi}$$

$$p_1(s) = -\frac{BD}{C}\tilde{\varphi} \quad (13a)$$

$$f_2(s) = \frac{1}{2!C^2}(BD-CA)^2\tilde{\varphi}$$

$$p_2(s) = \frac{1}{2!}A^2\tilde{\varphi} - f_2(s)$$

$$q_2(s) = \frac{1}{1\cdot 2}BDp_1(s) \quad (13b)$$

$$f_3(s) = \frac{1}{3!C^3}(BD-CA)^3\tilde{\varphi}$$

$$p_3(s) = -\frac{1}{3!}A^3\tilde{\varphi} - f_3$$

$$q_3(s) = \frac{1}{3}BDp_2 - \frac{A}{3}q_2$$

$$r_3(s) = \frac{1}{2\cdot 3}BDq_2 \quad (13c)$$

$$f_4(s) = \frac{1}{4!C^4}(BD-CA)^4\tilde{\varphi}$$

$$p_4(s) = \frac{1}{4!}A^4\tilde{\varphi} - f_4$$

$$q_4(s) = \frac{1}{4}BDp_3 - \frac{A}{4}q_3$$

$$r_4(s) = \frac{1}{2}\left[\frac{1}{4}BDq_3 - \frac{A}{4}(2r_3)\right]$$

$$s_4(s) = \frac{1}{3\cdot 4}BDr_3 \quad (13d)$$

$$f_5(s) = \frac{1}{5!C^5}(BD-CA)^5\tilde{\varphi}$$

$$p_5(s) = -\frac{1}{5!}A^5\tilde{\varphi} - f_5$$

$$q_5(s) = \frac{1}{5}BDp_4 - \frac{A}{5}q_4$$

$$r_5(s) = \frac{1}{2}\left[\frac{1}{5}BDp_4 - \frac{A}{5}(2r_4)\right]$$

$$s_5(s) = \frac{1}{3}\left[\frac{1}{5}BDr_4 - \frac{A}{5}(3s_4)\right]$$

$$t_5(s) = \frac{1}{4\cdot 5}BDs_4. \quad (13e)$$

Substituting equations (12) and (13) into equation (8) and arranging yields

$$\tilde{T}_a(x,y,s) = \tilde{\varphi}(s)\left\{e^{\eta} - e^{-Ax-Cy} \times \left[\sum_{k=1}^{\infty}\left(\frac{BDx}{C}\right)^k \cdot \frac{1}{k!}\sum_{n=0}^{k-1}(Cy)^n\right]\right\} \quad (14)$$

where

$$\eta = \frac{BD-CA}{C}x.$$

Substituting equation (14) into equation (7a) and rearranging yields the result of $\tilde{T}_b(x,y,s)$ as

$$\tilde{T}_b(x,y,s) = \tilde{\varphi}(s)\frac{D}{C}\left\{e^{\eta} + e^{-Ax-Cy} \times \left[\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{BDx}{C}\right)^k \cdot \sum_{n=0}^{k}(Cy)^n\right]\right\}. \quad (15)$$

Substitution of equations (14) and (15) into equation (6a) yields $\tilde{T}_w(x,y,s)$ as

$$\tilde{T}_w(x,y,s) = D\tilde{\varphi}(s)\left\{e^{\eta} - e^{-Ax-Cy} \times \left[\sum_{k=1}^{\infty}\left(\frac{BDx}{C}\right)^k \cdot \frac{1}{k!}\sum_{n=0}^{k-1}(Cy)^n\right]\right\} + B\tilde{\varphi}(s)\frac{D}{C} \times \left\{e^{\eta} + e^{-Ax-Cy}\left[\sum_{k=0}^{\infty}\left(\frac{BDx}{C}\right)^k \cdot \frac{1}{k!}\sum_{n=0}^{k}(Cy)^n\right]\right\}. \quad (16)$$

ASYMPTOTIC BEHAVIOUR

It is difficult to express equations (14)–(16) in terms of elementary functions of t analytically. However, there are simple asymptotic solutions. Expansions for small values of t can be deduced from the behaviour of expressions (14)–(16) for $|s| \gg 1$. Thus, the asymptotic values of T_a, T_b and T_w for small values of t are given as

$$T_a(x,y,t) \cong e^{-x}L^{-1}\left\{\tilde{\varphi}(s)\left(1 + \frac{BDx}{C}\right)\right\} - e^{-x-y}L^{-1}\left\{\frac{BDx}{C}\tilde{\varphi}(s)\right\} \quad (17a)$$

$$T_b(x,y,t) \cong [e^{-x} - e^{-x-y}]L^{-1}\{\tilde{\varphi}(s)D/C\} \quad (17b)$$

$$T_w(x,y,t) \cong e^{-x}\, L^{-1}\left\{\tilde{\varphi}(s) D\left(1+\frac{BDx}{C}\right)\right\}$$
$$-e^{-x-y}\, L^{-1}\left\{\frac{BD^2 x}{C}\tilde{\varphi}(s)\right\}$$
$$+[e^{-x}-e^{-x-y}]L^{-1}\{\tilde{\varphi}(s)BD/C\}. \quad (17c)$$

The above expressions are exact for small values of t, but they will not be of practical use when $t \gg 1$. The steady-state temperatures of both fluids and the core wall are determined by the behaviour of equations (14)–(16) in the neighbourhood of the origin in the complex domain. For $s \to 0$, equations (14)–(16) give

$$T_a(x,y,t) \cong \left\{1 - e^{-(Rx+y)/(1+R)} \sum_{k=1}^{\infty} \frac{1}{k!}\left(\frac{Rx}{1+R}\right)^k \cdot \sum_{n=1}^{k-1}\left(\frac{y}{1+R}\right)^n\right\} \lim_{s\to 0}\{\tilde{\varphi}(s)s\} \quad (18a)$$

$$T_b(x,y,t) \cong \left\{1 - e^{-(Rx+y)/(1+R)} \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{Rx}{1+R}\right)^k \cdot \sum_{n=0}^{k}\left(\frac{y}{1+R}\right)^n\right\} \lim_{s\to 0}\left\{\frac{D}{C}\tilde{\varphi}(s)s\right\} \quad (18b)$$

$$T_w(x,y,t) \cong \left\{1 - e^{-(Rx+y)/(1+R)} \sum_{k=1}^{\infty} \frac{1}{k!}\left(\frac{Rx}{1+R}\right)^k \cdot \sum_{n=1}^{k-1}\left(\frac{y}{1+R}\right)^n\right\} \lim_{s\to 0}\{D\tilde{\varphi}(s)s\} + \left\{1 - e^{-(Rx+y)/(1+R)} \cdot \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{Rx}{1+R}\right)^k \cdot \sum_{n=0}^{k}\left(\frac{y}{1+R}\right)^n\right\} \lim_{s\to 0}\left\{\frac{BD}{C}\tilde{\varphi}(s)s\right\}. \quad (18c)$$

RESULTS AND DISCUSSION

The numerical inversion of Laplace transforms proposed by Honig and Hirdes [8] can be applied to invert the transformed temperatures $\tilde{T}_a$, $\tilde{T}_b$ and $\tilde{T}_w$, shown in equations (14)–(16), to the results in the physical quantities. The method for the acceleration of convergence, such as the Shanks transformation [9], may be employed to calculate the sum of the series expansions, shown in equations (14)–(16), when they are slowly convergent. The explicit analytical expressions for $\tilde{T}_a(x,y,s)$, $\tilde{T}_b(x,y,s)$ and $\tilde{T}_w(x,y,s)$ have been processed for several meaningful choices of $\varphi(t)$ in order to simulate the most common transient operations. In particular the step response ($\varphi = 1$), ramp response ($\varphi = \alpha t$) and exponential response ($\varphi = \exp(\alpha t)$) will be considered. A much wider range of possible transient responses is covered by these results. In the illustrative examples, α is taken to be unity in the ramp and exponential responses. The parameters N_a and N_b are equal to 2. For the present problem Spiga and Spiga [2] obtained the analytical solutions and also showed the series representation of the exponential response for $R = 1$. However, their inversive work of the transformed temperatures is complex and difficult. Moreover, their analytical results are limited in the present heavier restrictions, as shown in equations (3).

It is seen that the series expansions in equations (14)–(16) are very simple and regular. This statement implies that the transformed results shown in equations (14)–(16) are easily expressed in a computational program even though the value of $\varphi(t)$ is arbitrarily chosen. Thus, their numerical calculations are very inexpensive in terms of computation time. Furthermore, the series representation of equations (14)–(16) can be applied to obtain the overall output of the gas-to-gas cross-flow heat exchangers for arbitrary choices of $\varphi(t)$ without any difficulty. The asymptotic values for $t \to \infty$ in the step response reproduced are in good agreement with previous results given by Baclic and Heggs [1] and Spiga and Spiga [2]. The exit temperatures of T_a and T_b corresponding to $R = 1$ and $N_a = N_b = 2$ are shown in Figs. 1–6 for the step, ramp

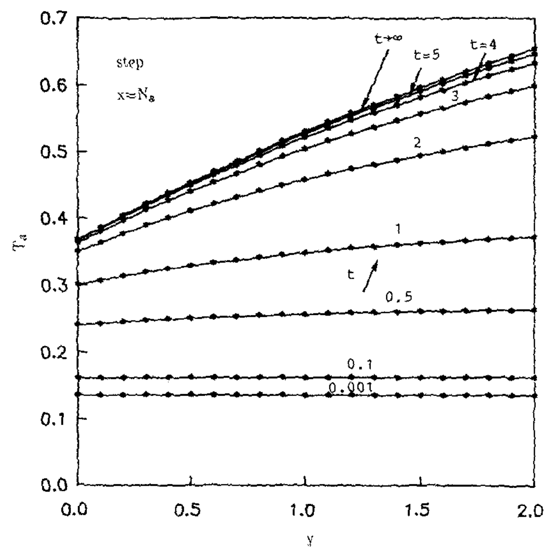

FIG. 1. Outlet temperature distribution of the primary fluid for a step response with $R = 1$.

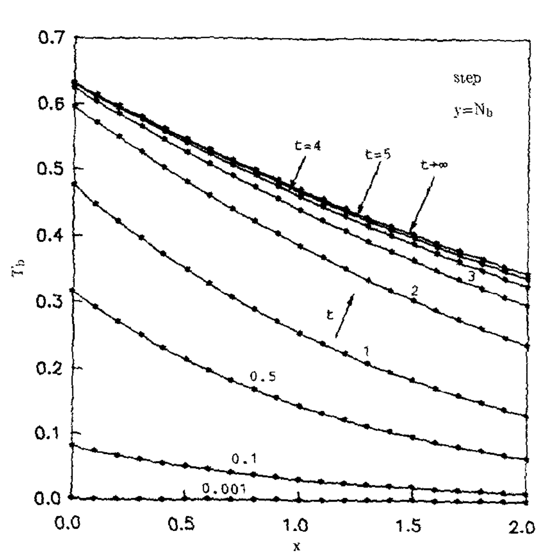

FIG. 2. Outlet temperature distribution of the secondary fluid for a step response with $R = 1$.

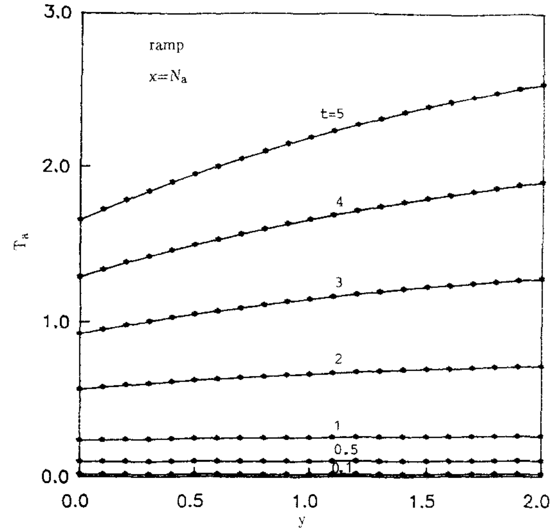

FIG. 3. Outlet temperature distribution of the primary fluid for a ramp response with $R = 1$.

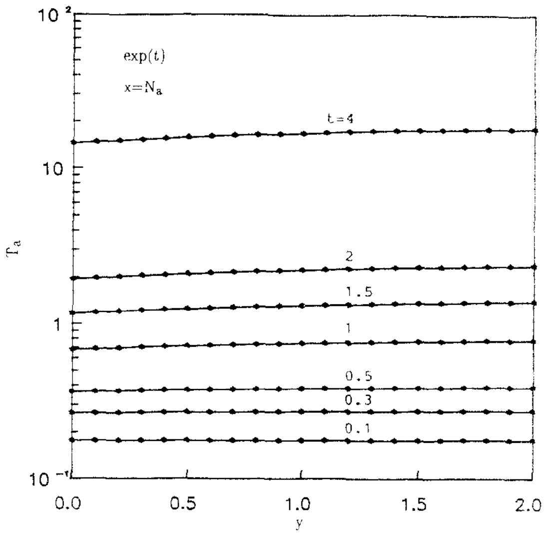

FIG. 5. Outlet temperature distribution of the primary fluid for an exponential response with $R = 1$.

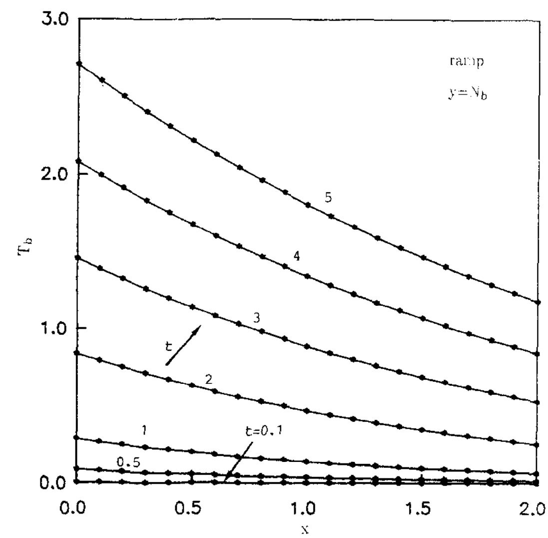

FIG. 4. Outlet temperature distribution of the secondary fluid for a ramp response with $R = 1$.

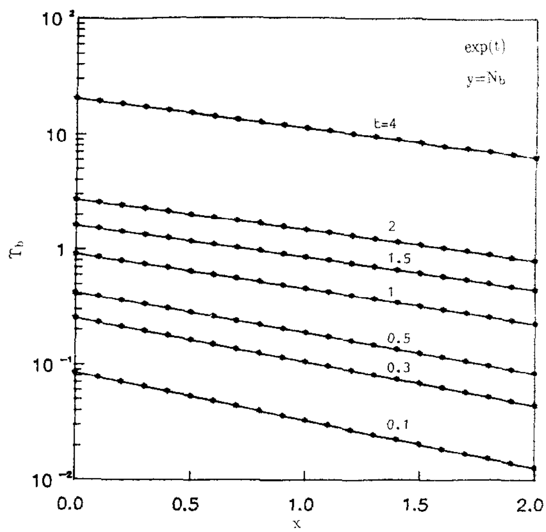

FIG. 6. Outlet temperature distribution of the secondary fluid for an exponential response with $R = 1$.

and exponential responses, respectively. The results for the step and ramp responses shown in Figs. 1–4 agree well with those contained in Figs. 7–10 of Spiga and Spiga [2]. This conclusion implies that the present method has good accuracy. The temperature of the core wall is not expressed in the present paper. However, it is not difficult to invert the transformed temperature of the core wall $\tilde{T}_w(x, y, s)$, as shown in equation (18c), to the physical quantity by using the numerical inversion of the Laplace transform [8] if $T_w(x, y, t)$ is required.

The exit temperatures of T_a and T_b for the step response for $R = 0.5$ and 2 are respectively shown in Figs. 7–10. Comparisons for these cases are impossible because there are no available data in the literature. However, it is seen that higher values of R imply smaller $(mc)_a$ and higher $(hA^*)_b$ for the fixed values of N_a, N_b, $(hA^*)_a$ and $(mc)_b$. Thus, the temperature at the outlet positions of both fluids will be increased with decreasing the value of R for $N_a = N_b = 2$. These results can be found from Figs. 7 to 10. This conclusion implies that the temperature response of $R = 0.5$ is faster approaching the steady state than that of $R = 2$. To avoid duplication, the effect of R on the transient behaviours of T_a and T_b for the ramp and exponential responses will not be investigated.

CONCLUSION

The two-dimensional transient response of gas-to-gas cross-flow heat exchangers is investigated ana-

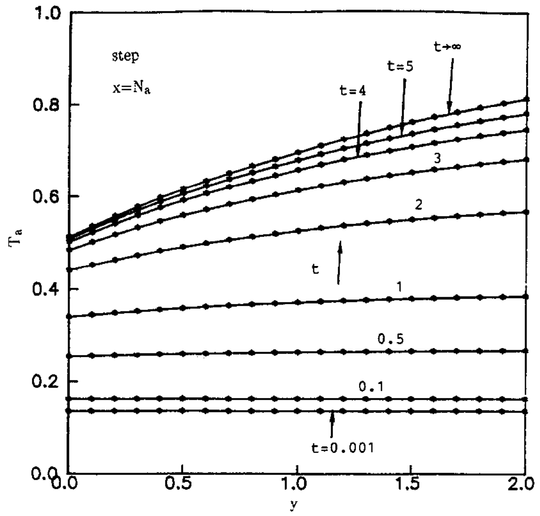

FIG. 7. Outlet temperature distribution of the primary fluid for a step response with $R = 0.5$.

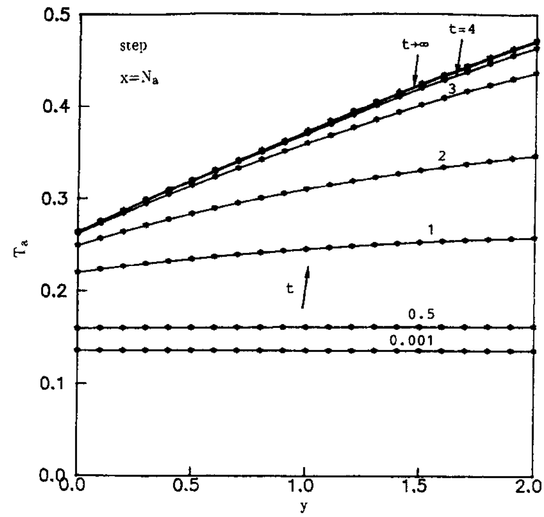

FIG. 9. Outlet temperature distribution of the primary fluid for a step response with $R = 2$.

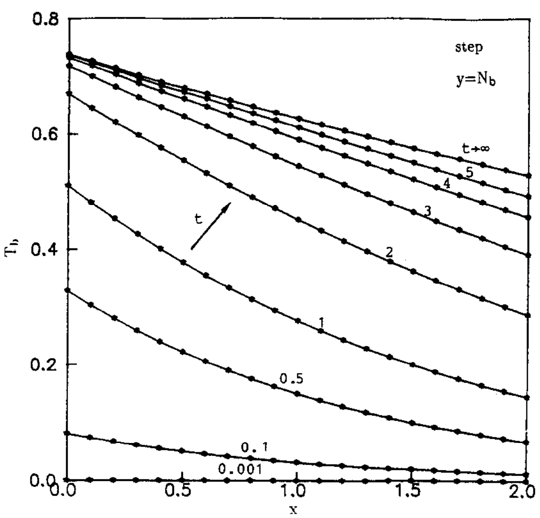

FIG. 8. Outlet temperature distribution of the secondary fluid for a step response with $R = 0.5$.

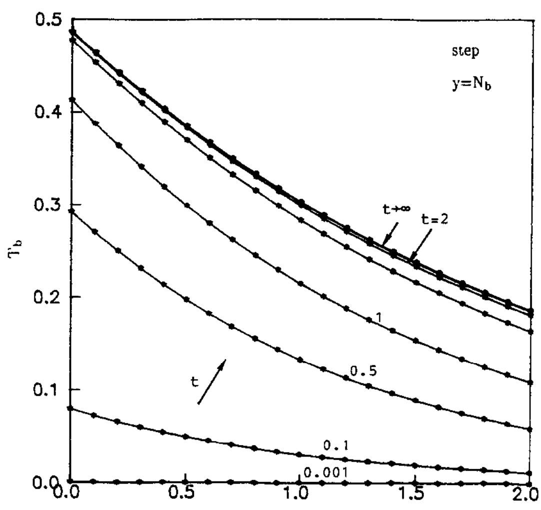

FIG. 10. Outlet temperature distribution of the secondary fluid for a step response with $R = 2$.

lytically. The method of the single Laplace transform method with respect to time in conjunction with the power series technique is applied to obtain the numerical solutions of such a problem. It is seen that the application of this technique to such problems is simple and can determine $\tilde{T}_a$, $\tilde{T}_b$ and $\tilde{T}_w$ with regular forms. Thus, the transformed temperatures are easily expressed in a computational program for arbitrary choices of $\varphi(t)$ and R without any difficulty. Moreover, the present numerical calculations are also very inexpensive in terms of computational time, and the temperature responses of the core wall and both fluids, at a specific time and position, can also be calculated. The present results are compared with the analytical results of Spiga and Spiga [2]. The present method shows satisfactory results for the present problem. It can be concluded that the present method has good accuracy and efficiency.

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METHODE SIMPLE POUR LA REPONSE VARIABLE D'ECHANGEUR DE CHALEUR A COURANTS CROISES GAZ–GAZ SANS AUCUN MELANGE

Résumé—On étudie analytiquement la réponse variable bidimensionnelle des échangeurs de chaleur à courants croisés gaz–gaz en utilisant la méthode de la transformée de Laplace pour des variations temporelles arbitraires de la température d'entrée du fluide primaire. Des solutions analytiques pour la distribution de la température transformée de la paroi et de chaque fluide sont présentées sous la forme d'une série de puissances avec le rapport des capacités thermiques, le nombre d'unités de transfert, la résistance au transfert thermique et les rapports des débits calorifiques des fluides. Toutes les températures transformées sont aisément inversées pour obtenir les grandeurs physiques en utilisant le schéma de l'inversion numérique de la transformée de Laplace. Comparée à d'autres résolutions analytiques, la méthode présente a bonne efficacité et précision.

EIN EINFACHES VERFAHREN ZUR BERECHNUNG DES DYNAMISCHEN VERHALTENS EINES GAS-KREUZSTROMWÄRMEÜBERTRAGERS OHNE QUERVERMISCHUNG

Zusammenfassung—Das zweidimensionale Übergangsverhalten eines Gas/Gas-Kreuzstromwärmeübertragers wird analytisch untersucht. Dabei wird das Verfahren einer einzelnen Laplace-Transformation für beliebige zeitliche Änderungen der anfänglichen Fluid-Eintrittstemperatur angewandt. Für die transformierten Temperaturverteilungen an den inneren Wänden und in beiden Fluiden werden analytische Lösungen in Gestalt eines Potenzansatzes angegeben, abhängig vom Verhältnis der Wärmekapazitäten, von *NTU*, vom Wärmeübergangswiderstand und vom Verhältnis der Wärmekapazitätsströme. Die transformierten Temperaturen für die Wände und die beiden Fluide können leicht unter Verwendung eines numerischen Inversionsverfahrens der Laplace-Transformation in den physikalischen Bereich zurücktransformiert werden. Im Vergleich zu anderen analytischen Lösungen zeigt das vorgestellte Verfahren hohe Genauigkeit und Effizienz.

ПРОСТОЙ МЕТОД ОПЯЕДЕЛЕНИЯ ПЕРЕХОДНОЙ ХАРАКТЕРИСТИКИ ГАЗО–ГАЗОВЫХ ПЕРЕКРЕСТНЫХ ТЕПЛООБМЕННИКОВ

Аннотация—Методом однократного преобразования Лапласа аналитически исследуется двумерная переходная характеристика газо–газовых перекрестных теплообменников в случае произвольных изменений температуры первичного теплоносителя на входе. Аналитические решения для изображений температурых распределений стенки и обеих рабочих сред представлены в виде степенных рядов, включающих отношения теплоемкостей и сопротивление теплопереносу. Эти соотношения в изображениях легко обращаются с использованием численной схемы обращения преобразования Лапласа. По сравнению с другими аналитическими решениями предложенный метод обладает высокой точностью и эффективностью.